$D+\alpha$ operator and related problems in physics.

Pablo Moreira-Galván
Universidad Anáhuac Querétaro

The three-dimensional Moisil-Teodorescu differential operator D is defined by

$$D = e_1\partial_1 + e_2\partial_2 + e_3\partial_3,$$

where $\partial_k = \partial/\partial x_k$, $k = 1, 2, 3$. It is known that the action of the operator D to any differentiable function $w = w_0 + \vec{w}$ can be written as

$$Dw = -\operatorname{div}\vec{w} + \operatorname{grad} w_0 + \operatorname{curl}\vec{w},$$

With this we are able to find different representations to classical equations

- A Beltrami Field is a function such that

  $$\operatorname{curl}\vec{w} + \lambda\vec{w} = 0, \quad \text{in } \Omega. \tag{1}$$

  This problems is equivalent to solve

  $$(D+\lambda)w = 0 \tag{2}$$

  which in [1] find explicit solutions.

- The Helmholtz equation $\Delta + \lambda^2$ can be decomposed as follows (see [2])

  $$\Delta + \lambda^2 = -(D+\lambda)(D-\lambda) \tag{3}$$

- Maxwell's equations for time-harmonic electromagnetic fields in a chiral medium have the form

  $$\begin{aligned}&\operatorname{div}\widetilde{E}(x) = \operatorname{div}\widetilde{H}(x) = 0, \;\; \operatorname{curl}\widetilde{E}(x) = i\omega\widetilde{B}(x)\\ &\operatorname{curl}\widetilde{H}(x) = -i\omega\widetilde{D}(x),\end{aligned}$$

  with the constitutive relations (see [2]) and using the change of variables

  $$\vec{\zeta}(x) = \vec{E}(x) + i\vec{H}(x), \;\; \vec{\eta}(x) = \vec{E}(x) - i\vec{H}(x).$$

  Is equivalent to

  $$\left(D + \frac{\alpha}{1+\alpha\beta}\right)\vec{\zeta}(x) = 0, \left(D - \frac{\alpha}{1-\alpha\beta}\right)\vec{\eta}(x) = 0. \tag{4}$$

  We constructed an invertible quaternionic integral operator which transforms solutions of the operator D into solutions these equations.

References

[1] B. Delgado & P. Moreira *On the Construction of Beltrami Fields and Associated Boundary Value Problems* Applied Clifford Algebras 2024.

[2] V.V Kravchenko *Applied Quaternionic Analisis.* Heldermann Verlag, 2003.