

# Generalized separation of variables and Nijenhuis geometry

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Let  $M^n$  be a manifold, equipped with metric  $g$  and potential function  $U$ . The corresponding geodesic flow is given on  $T^*M^n$  via Hamiltonian

$$\frac{1}{2}g^{ij}p_ip_j + U.$$

. It is a well-known fact that if such flow admits an orthogonal separation of variables, then the separation coordinates are given by a certain Nijenhuis operator with real simple spectrum.

It turns out that this relation with Nijenhuis geometry is much deeper — one can construct an analog integrable system not only for the aforementioned operators, but for operators containing Jordan blocks. This is done via a simple algebraic construction.

This relation not only provides integrable systems (which are, in some sense, close neighbors of well-known separable systems), but also allows one to solve some fundamental problems in the theory of projectively equivalent metrics.