Non-integrability of the n-body problem

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We consider the classical planar n body problem. In the center of mass frame. It admits the energy and the angular momentum first integrals. The integrability of the system restricted to a common level of these first integrals is investigated. We show that if $n > 2$, the system is not integrable on all but one level corresponding to the zero value of the energy and angular. In our proof we use differential Galois methods. We also discuss the applicability of our approach to study the integrability of charged n body problem.

References

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