Non-compactness effects in integrable systems: examples and results

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The topological classification of integrable Hamiltonian systems, constructed and developed by A.T. Fomenko, his scientific school and co-authors, was applied to a wide class of mechanical and physical problems. An important assumption of many statements of the theory is the compactness of the fibers of the Liouville foliation.

In systems with non-compact foliations, new effects arise: e.g. the flows of Hamiltonian vector fields can become incomplete, and the preimage of the bifurcation value does not necessarily contain critical points (in particular, it can be empty). We will briefly discuss several well-known systems with non-compact foliations, as well as the results obtained by various authors on their topological analysis and the problem of classification. We note here the review by A.T.Fomenko and D.A.Fedoseev on systems with non-compact foliations (2020, J. Math. Sc.).

Pseudo-Euclidean analogues of integrable mechanical systems (see A.~Borisov, I.~Mamaev, 2016) turn out to be an important class of such systems. New results on topology of Liouville foliations of pseudo-Euclidean Euler, Lagrange and Kovalevskaya tops, Zhukovsky and Klebsch systems will be presented. Both compact and non-compact fibers, their bifurcations (including non-critical one) appear in such systems. Bifurcations and Liouville foliations bases (analogs of Fomenko graphs) are also determined. Several results on non-compactness effects in billiard systems also will be discussed

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