On classification of regular semisimple algebraic Nijenhuis operators

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Abstract

Algebraic Nijenhuis operators naturally appear in the study of integrable systems. They can be described as linear operators L on a Lie algebra $\mathfrak{g}$ satisfying the identity

$$L[L\xi, \eta] + L[\xi, L\eta] - [L\xi, L\eta] - L^2[\xi, \eta] = 0$$

for arbitrary $\xi, \eta \in \mathfrak{g}$, where $[\,,]$ is the commutator in $\mathfrak{g}$.

For the case when such an algebraic Nijenhuis operator on a Lie algebra $\mathfrak{g}$ is regular semisimple, i.e., it has an eigenbasis with pairwise different eigenvalues, there is an exellent statement due to Y. Kosmann-Schwarzbach and F. Magri which says that each pair of eigenvectors from this basis generates a two-dimensional subalgebra in $\mathfrak{g}$. Moreover, the inverse procedure also takes place, namely, the presence of a basis in a Lie algebra $\mathfrak{g}$ with such properties makes it possible to construct a regular semisimple algebraic Nijenhuis operator on $\mathfrak{g}$. Such bases in Lie algebras are called Nijenhuis eigenbases.

There are examples of Lie algebras possessing Nijenhuis eigenbases, but it is not clear how to describe all Lie algebras with such bases. Besides that, different bases can generate equivalent Nijenhuis operators. The aim of the present talk is to propose some approach to description of Nijenhuis bases themselves, without thinking about which Lie algebras they define. In particular, it will be demonstrated how one can obtain all (non-equivalent) algebraic Nijenhuis operators in small dimensions by using the proposed approach.