

Exact quasi-periodic solutions to the KdV, sine(sinh)-Gordon, and \pm mKdV equations

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ABSTRACT. Exact quasi-periodic solutions of the KdV, sine(sinh)-Gordon, and mKdV (focusing and defocusing) equations are expressed via multiply periodic \wp -functions. These solutions are obtained by algebraic integration of hamiltonian systems from the corresponding hierarchies. The KdV, sine(sinh)-Gordon, and \pm mKdV hierarchies all possess spectral curves which are hyperelliptic, with one branch point at infinity. Such a spectral curve of an arbitrary genus g is uniformized by the associated \wp -functions, which generalize the Weierstrass \wp -function. The \wp -functions are used to express quasi-periodic solutions to g -gap systems. Solutions to the KdV, and sine(sinh)-Gordon hierarchies were known before, and solutions to the mKdV hierarchies (focusing and defocusing) are new. The problem of reality conditions for all mentioned hierarchies is revised based on the knowledge of behavior of \wp -functions, and an accurate solution is obtained, which is also a new result. The proposed solutions are illustrated by plots in genera 2 and 3.

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